Optimal Experiment Design for Thermal Characterization of Functionally Graded Materials

Kevin D. Cole
Mechanical Engineering Department
University of Nebraska—Lincoln

Outline

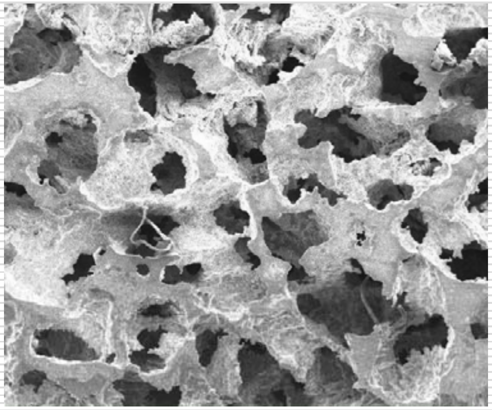
- Motivation: Design experiments to measure thermal properties of functionally-graded (FG) materials.
- Simulation studies of transient heating:
  - In FG material with k(x), conduction only;
  - In high-porosity material with large ΔT, both conduction and radiation present.
- Summary and future work.

Functionally-graded (FG) materials

- Material (or structure) with properties that vary though the thickness
- Property variations are of interest to improve thermal/stress performance
- FG materials include composites, built-up structures, metal foams, or any structure with variations designed into the material

Fabrication Methods for FG Materials

- Layer processing
  - Mechanical lamination
  - Spraying
  - Vapor deposition
- Bulk processing
  - Powder mixing and sintering
  - Fibers
- Diffusion processing
- Melt processing

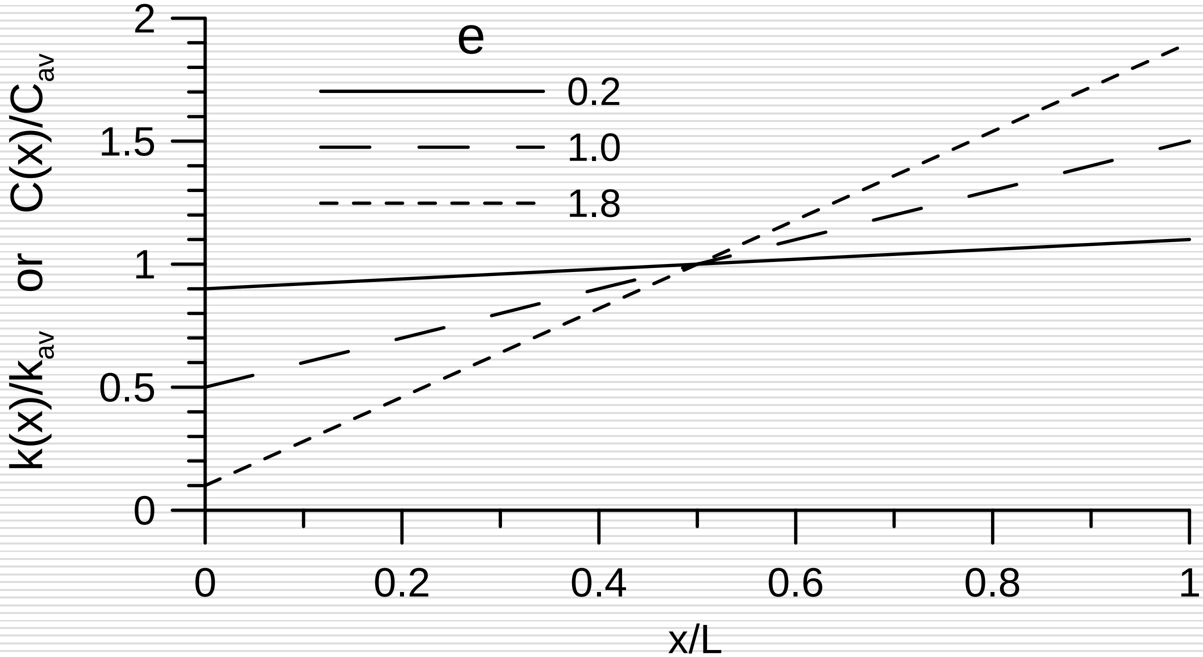


Example: porous metal foam with spatially-varying porosity.

Motivation for current project

- Functionally graded (FG) materials are being studied for thermal insulation (for aero-thermal protection or cryogenic tanks)
- Once fabricated, does a delivered material meet the specifications?
- Seek procedure to choose an experiment to measure spatially-varying thermal properties of FG materials (accurate; non-invasive; cost effective)

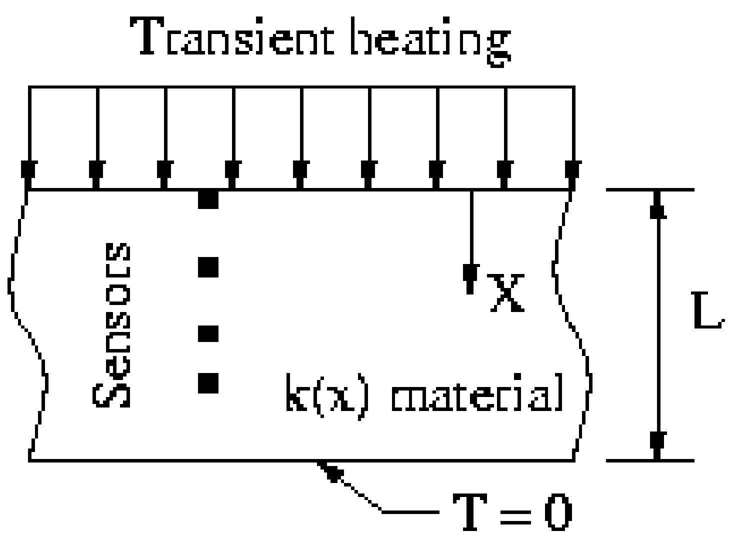
FG Material, linear variation of k(x)



- $k(x) = k_{av} [1 + e(x/L - 0.5)]$ (W/m/K)
- $C(x) = C_{av} [1 + e(x/L - 0.5)]$ (J/m^3/K)

Transient experiment for thermal property measurement.

- On-off heating on one side
- Fixed temperature on other side
- One or more temperature sensors
- Collect data during heating and continue while unheated.
- Analyze data for desired properties with parameter estimation.



Parameter Estimation

- Goal: given experimental data Y_j (θ_i), find parameters (k_{av}, C_{av}, e).
- Need model: given (k_{av}, C_{av}, e), compute $T_j(\theta_i)$. (j … sensor; i … time)
- Minimize the sum-of-square of error

$$[\ Y_j(\theta_i) - T_j(\theta_i)\ ]^2$$

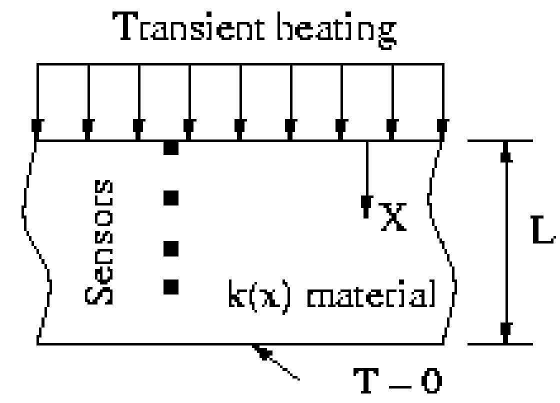
- Minimization requires derivatives of the model temperatures with respect to the parameters (sensitivity coefficients).

Computational Model
(solution method: finite difference)

$$\frac{\partial}{\partial x^+}\left(k^+\frac{\partial T^+}{\partial x^+}\right) = C^+\frac{\partial T^+}{\partial \theta}; \qquad 0 < x^+ < 1$$

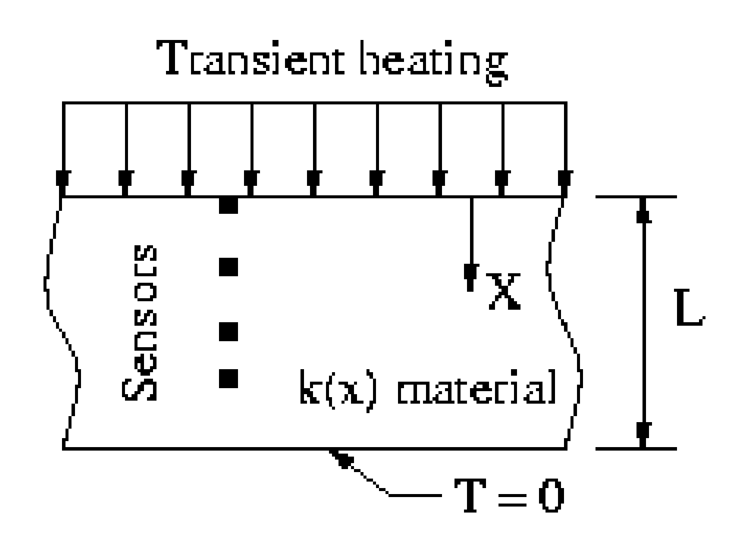
$$\text{at } x^+ = 0, \; -k^+(0)\frac{\partial T^+}{\partial x^+} = \begin{cases} 1, & \theta < \theta_h \\ 0, & \theta > \theta_h \end{cases}$$

$$\text{at } x^+ = 1, \; T^+(1,\theta) = 0$$
$$\text{at } \theta = 0, \; T^+(x^+,0) = 0$$



Seek best experiment by varying:

- Number and location of sensors
- Duration of heating period
- Duration of entire data record
- Location of heating (x=0 or x=L)
- Apply to several different spatial variations (slope, e)



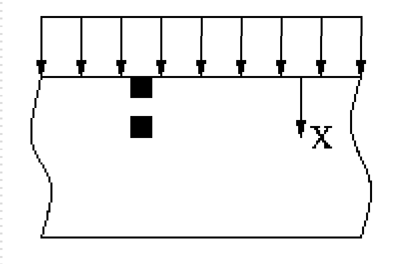
Compare 2 Experimental Designs

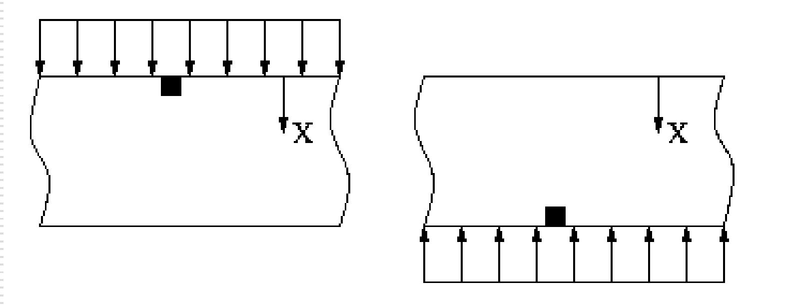
Experiment 1:
Heat at x=0
with two sensors:

- Sensor at x=0.
- Sensor at x=L/4.

Experiment 2:
Two heating events
with one sensor each:

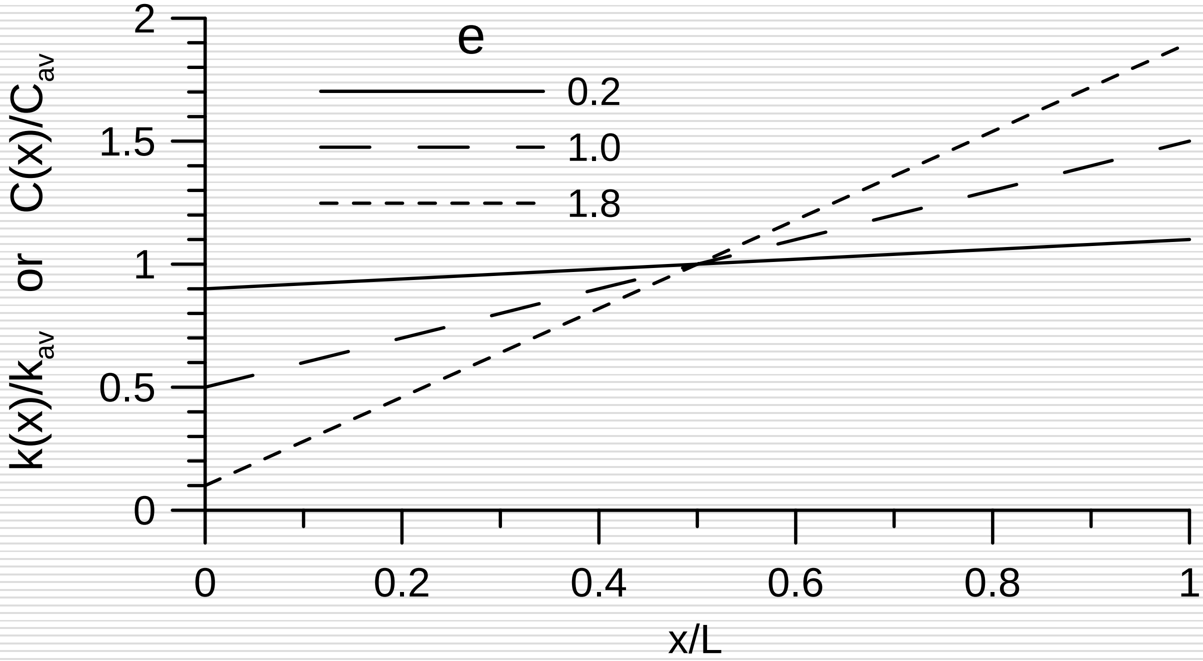
- Heat/sensor at x=0.
- Heat/sensor at x=L.





Seek optimum for each experimental design by varying heat duration and data duration, for several values of slope e.

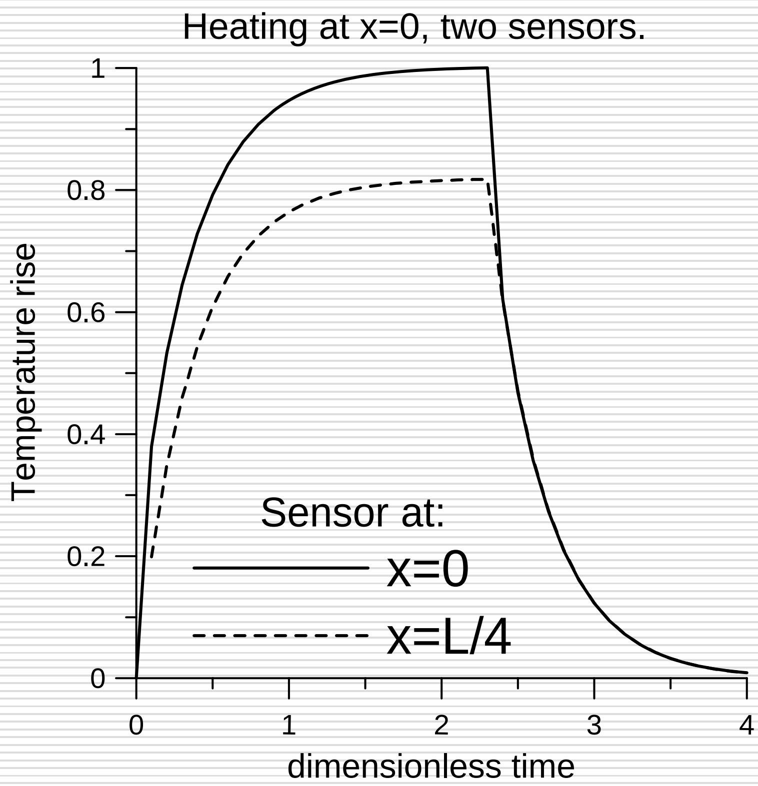
FG Material, linear variation of k(x)



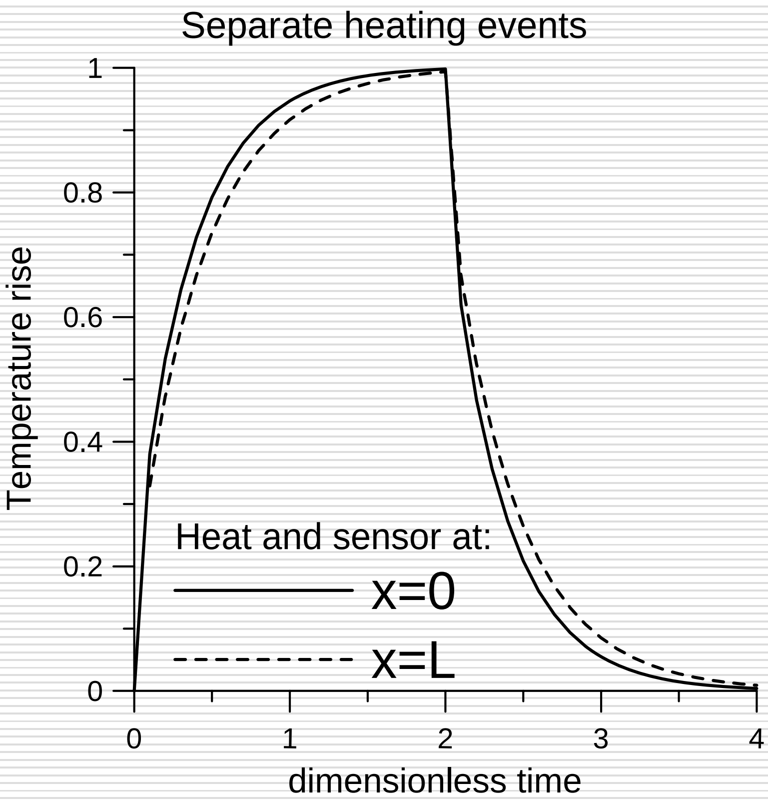
- $k(x) = k_{av}\ [1 + e(x/L - 0.5)]$ (W/m/K)
- $C(x) = C_{av}\ [1 + e(x/L - 0.5)]$ (J/m^3/K)

Calculated temperatures (e = 0.2)

Experiment 1.



Experiment 2.



Sensitivity coefficients

- Minimization of error requires sensitivity coefficients (derivatives wrt parameters b_k)

$$X_{jk}(i) = b_k \frac{\partial T_j^+}{\partial b_k}$$

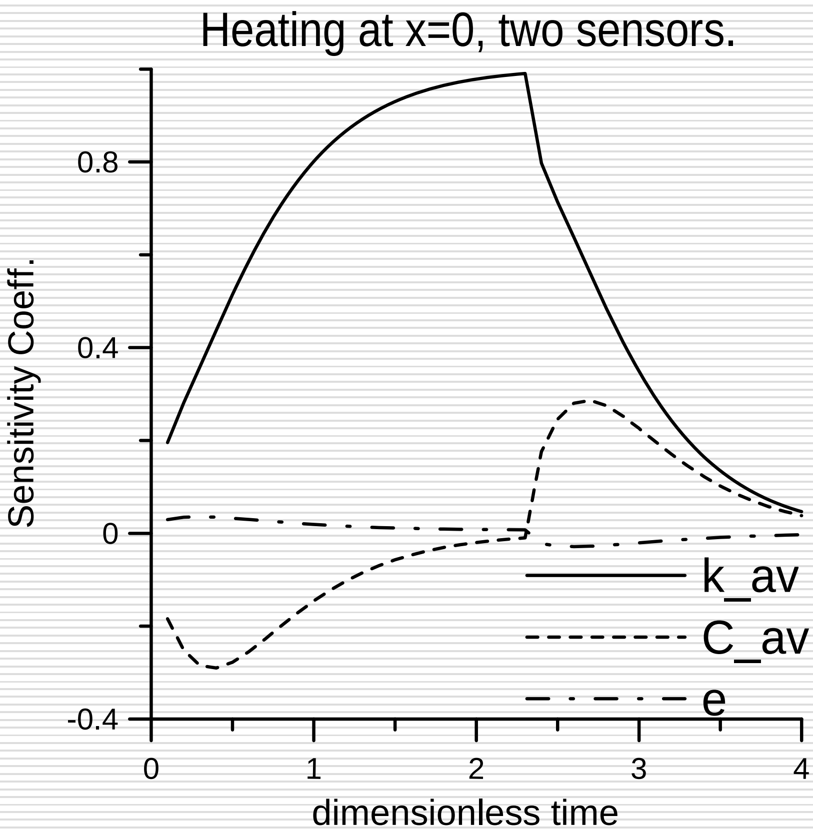
- which are computed from the model:

$$X_{jk}(i) \approx b_k \frac{[T_{ij}^+((1+\epsilon)b_k) - T_{ij}^+(b_k)]}{\epsilon b_k}$$

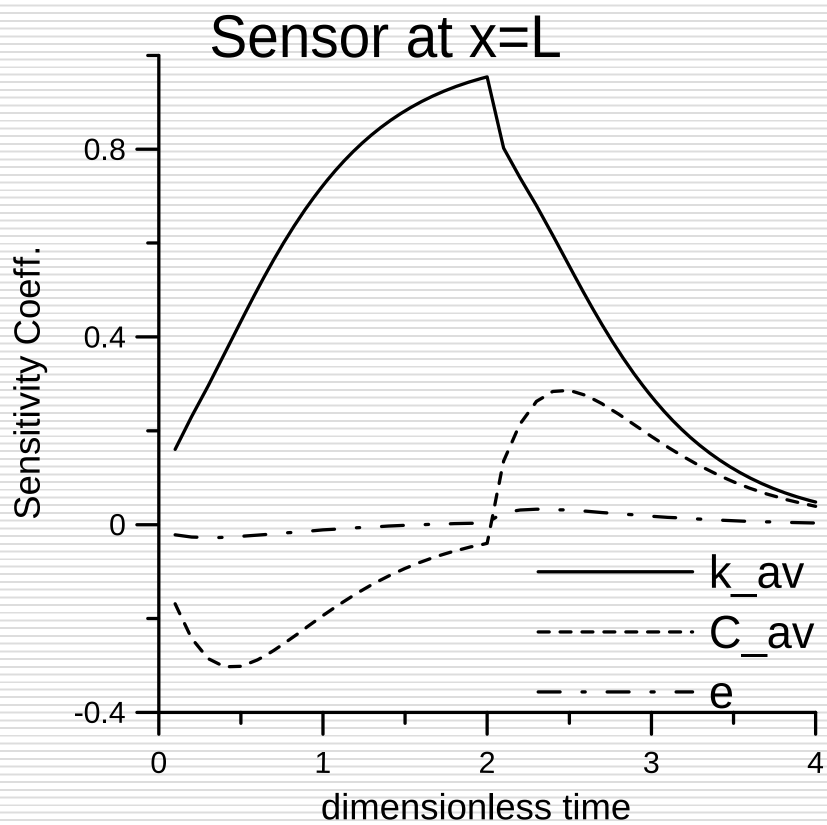
- and assembled into matrix **X**.

Sensitivity Coefficients (e = 0.2)

Experiment 1.



Experiment 2.



Sensitivities and Optimality

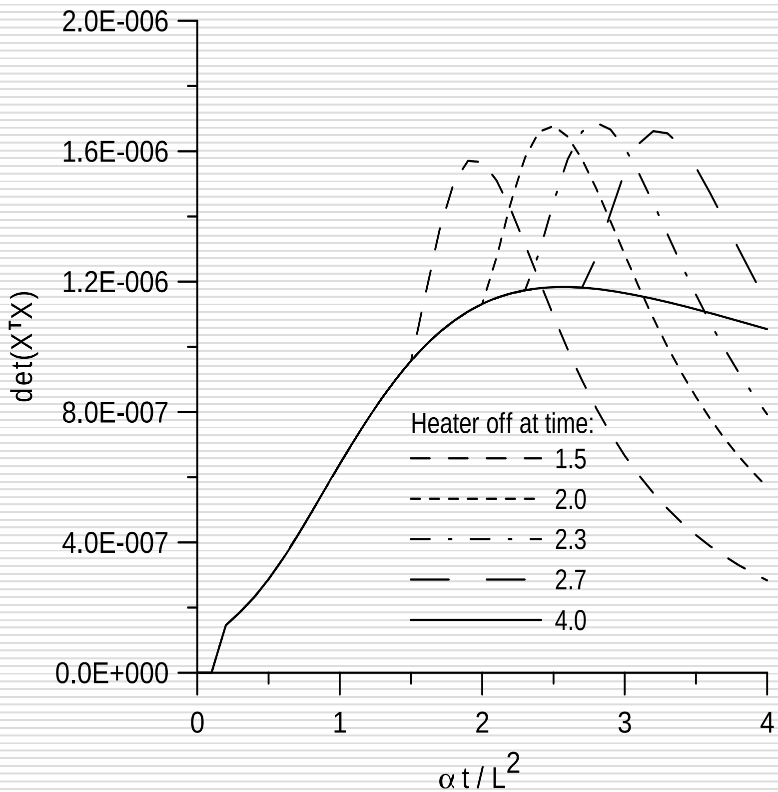
- Sensitivities should be as large as possible.
- Sensitivities must be linearly independent to estimate two or more parameters.
- Given sensitivity matrix **X**, the experiment with the "largest" and "most independent" sensitivities occurs at the maximum value of the determinant of matrix $\mathbf{X}^T\mathbf{X}$:

$$D = \frac{1}{s\ n\ (T^{+}_{max})^2} det(\boldsymbol{X}^T \boldsymbol{X})$$

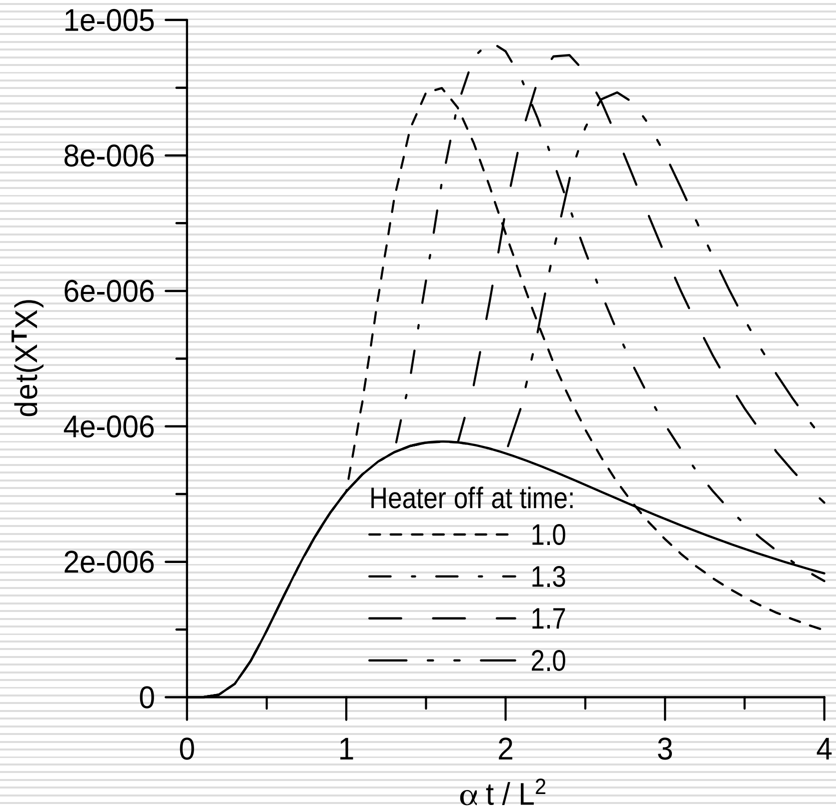
(s - sensors; n - time steps;)

Optimality Criterion (for e = 0.2), vary heating duration.

Experiment 1.
Sensors x=0, x = L/4



Experiment 2.
Two heating events



Optimal Experiments.

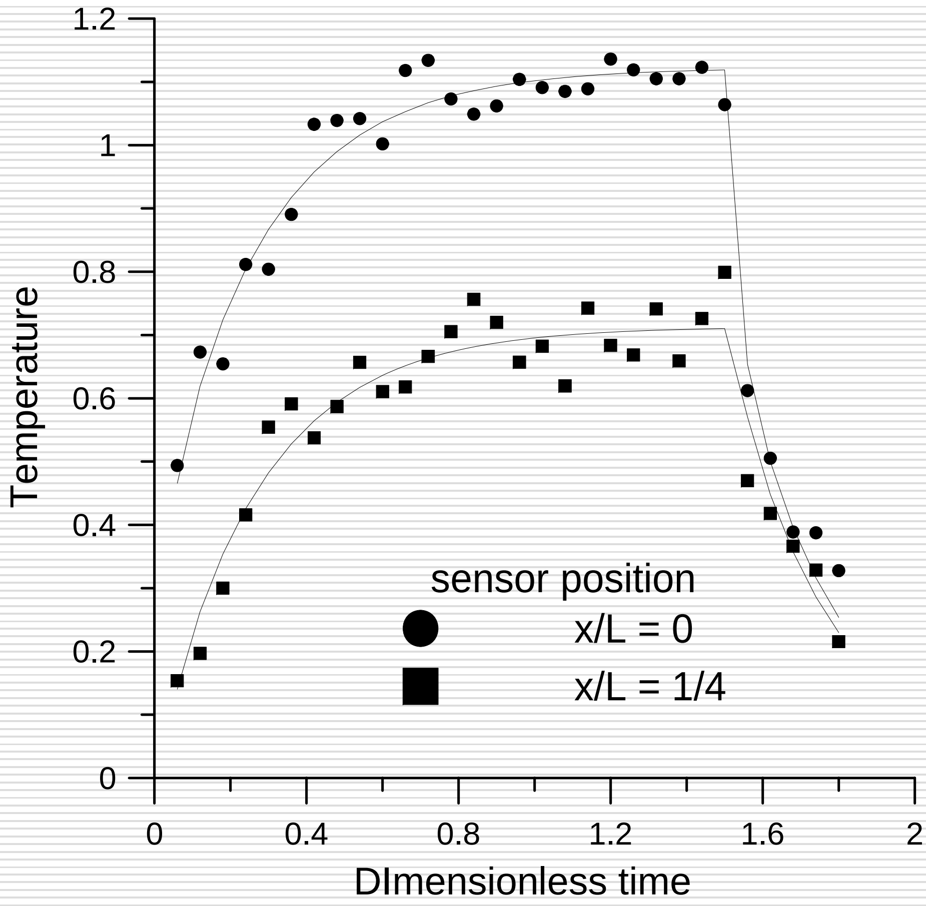
Experiment Design	e	Heat time	Data time	Max. **D** value
1. Heat x=0, sensors at x=0, x=L/4.	0.2 1.0 1.8	2.3 1.5 1.5	2.8 1.8 1.7	1.7 E-06 8.7 E-05 1.2 E-03
2. Two heat events, one sensor each.	0.2 1.0 1.8	1.3 1.7 1.3	1.9 2.2 1.5	9.5 E-06 24. E-05 2.2 E-03
3. k = const.	--	2.25	3.0	2.0 E-01

Simulated data analysis

- Compute T_{exact} vs time from simulation of optimal experiment
- Add noise: $T_{data} = T_{exact} + \varepsilon_i$

  ε_i= Zero mean, gaussian, additive error
- Analyze T_{data} by a curve fit to the simulation to estimate parameters k_{av}, c_{av}, e.

Simulate Experiment 1, two sensors, error 5% variance



	Exact values	Curve fit
k_{av}	1.0	1.0148
c_{av}	1.0	1.0080
e	1.0	1.0388

Analysis of optimum experiments (one heat event and two sensors)

slope e	Error variance	% error in k_{av}	% error in c_{av}	% error in e
0.2	1%	0.22	0.92	13.9
1.0	1%	0.41	0.79	0.27
1.8	1%	0.39	0.14	0.14
0.2	5%	2.68	7.13	*
1.0	5%	1.48	0.80	3.88
1.8	5%	0.42	3.00	0.33

* No convergence

Discuss optimal experiments for FG materials with k(x) linear

- Best experiment combines two heat events, heat/sensor at x=0 and at x=L. No interior sensors are needed, non-destructive.
- Length of data record is critical, as optimality criterion D=det($\mathbf{X}^T\mathbf{X}$) has a very narrow peak.
- Optimal heat duration and data record duration vary somewhat with spatial-variation slope, *e*.
- Possible to fit three parameters (k_{av}, c_{av}, e) with data from two sensors.
- More accurate parameter estimates are possible for materials with larger variation in k (larger *e*); it is easier to "see" larger-*e* values.

Summary

- Design of experiments for finding thermal properties of a functional-graded material with k(x) across the thickness.
- Sensitivity to parameters is used to construct optimality criterion det($\mathbf{X}^T\mathbf{X}$) for measurement of thermal properties.
- Applied to material with linear variation of k(x).
- Presented results of simulated data analysis.

Future work.

- Investigate additional materials
  - Other k(x) distributions (conduction only)
  - Metal foams (conduction and radiation)
- Automate the search for largest value of optimality criterion $\det(X^T X)$, to determine the best experimental conditions.
- Is the optimum D-value large enough? Simulate additional cases to estimate parameters from noise-containing data.
- Perform an actual experiment to obtain data on a suitable material; analyze laboratory data.

Acknowledgement

The work was carried out with help from

- Paul Crittenden, post doc
- Imran Amirani, M.S. student
- Ryan Sneed, undergraduate student

Funding provided by:

- NASA Langley grant NAG-1-01087, contract supervised by Max Blosser
- University of Nebraska